MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2016/2017

DIM5068 – MATHEMATICAL TECHNIQUES 2

(Diploma in Information Technology)

10 MARCH 2017
9.00 a.m – 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **2** pages with **4** questions. Key formulae are given in the Appendix.
2. Answer **ALL** questions.
3. Write your answers in the answer booklet provided.
4. All necessary working steps must be shown.

QUESTION 1

a. Simplify $\frac{5-\sqrt{2}i}{7-\sqrt{2}i}$ and leave your answer in standard form. (6 marks)

b. Given $3x^2+15=10x$.

i. Compute the discriminant. (2 marks)

ii. Determine the complex zeros of the polynomial. (3 marks)

iii. Write the solutions in factored form. (2 marks)

c. Given that $\tan\theta=\frac{1}{\sqrt{3}}$, write the complex number, **z**, in polar form. (5 marks)

d. Let the KLCC be at the origin in a complex plane. If a tourist is standing $\frac{4\pi}{3}$ from the initial side of the plane, compute the position of the tourist from KLCC in a standard complex number. (7 marks)

[TOTAL: 25 MARKS]

QUESTION 2

a. Given $y=\frac{\sin(e^{2x})}{x^3}$, find $\frac{dy}{dx}$. [Hint: use Quotient Rule] (6.5 marks)

b. Differentiate $3x^2-5xy+y^2=10$ by using **implicit differentiation**. (5.5 marks)

c. Find the following integrals:

i. $\int\left(x^3+3x\right)^4\left(x^2+1\right)dx$ [Hint: use Substitution Rule] (6 marks)

ii. $\int_1^2 16\,t^3\ln t\ dt$ [Hint: use Integration by Part] (7 marks)

[TOTAL 25 MARKS]

Continued...

QUESTION 3

a. Solve the differential equation $\frac{dy}{dx} = \frac{(x^2+1)}{y^2}$ by using **separable method.**

(5 marks)

b. Given the differential equation $\frac{dy}{dx} - \frac{2y}{x} = 3x^3$.

i. Identify the $p(x)$ and $q(x)$. (1 mark)

ii. Calculate the **integrating factor** (μ). (2 marks)

iii. Find y given that $\mu y = \int \mu q(x)\,dx$. (4 marks)

c. Solve the initial value problem $y^{(3)} = \sin x + \cos x + 4$ given $y''(0) = y'(0) = 0$ and $y(0) = 1$. (13 marks)

[TOTAL 25 MARKS]

QUESTION 4

a. Given the following vector $\boldsymbol{a} = <1, 2, 3>$, $\boldsymbol{b} = <3, 2, 1>$ and $\boldsymbol{c} = <x, 2, z>$.

i. Find the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$. (5 marks)

ii. Compute $\boldsymbol{b} \cdot (-2\boldsymbol{a})$. (4 marks)

iii. Find the values of x and z for equation $3\boldsymbol{b} - 2\boldsymbol{c} = \boldsymbol{a}$ (4 marks)

b. Saripah wants to design a card with a triangular shape. Given that the vertices of the triangle are $P = (1, 2, 3)$, $Q = (-2, 4, 1)$, and $R = (2, 1, 1)$.

i. Determine $\vec{PQ}$ and $\vec{PR}$. (2 marks)

ii. Calculate the cross product of $\vec{PQ}$ and $\vec{PR}$. (3 marks)

iii. Compute the total area of the card. Round your answer to 2 decimal points. (3 marks)

c. Find the parametric equations of the line that goes through the points $P(1, 3, 2)$ and $Q(3, -1, 3)$. (4 marks)

[TOTAL 25 MARKS]

End of Page.

APPENDIX

Derivatives: $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Differentiation Rules

General Formulae

1. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$
2. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
3. $\frac{d}{dx}(x^n) = nx^{n-1}$
4. $\frac{d}{dx}[f(u)] = f'(u).\frac{du}{dx}$

Exponential and Logarithmic Functions

1. $\frac{d}{dx}(e^x) = e^x$
2. $\frac{d}{dx}(a^x) = a^x \ln a$
3. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
4. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

Trigonometric Functions

1. $\frac{d}{dx}(\sin x) = \cos x$
2. $\frac{d}{dx}(\cos x) = -\sin x$
3. $\frac{d}{dx}(\tan x) = \sec^2 x$
4. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
5. $\frac{d}{dx}(\sec x) = \sec x \tan x$
6. $\frac{d}{dx}(\cot x) = -\csc^2 x$

Table of Integrals

1. $\int u\,dv = uv - \int v\,du$
2. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{du}{u} = \ln|u| + C$
4. $\int e^u du = e^u + C$
5. $\int \sin u\,du = -\cos u + C$
6. $\int \cos u\,du = \sin u + C$
7. $\int \sec^2 u\,du = \tan u + C$
8. $\int \csc^2 u\,du = -\cot u + C$
9. $\int \sec u \tan u\,du = \sec u + C$
10. $\int \csc u \cot u\,du = -\csc u + C$

Application of Integrals:

Areas between Curve, $A = \int_a^b [f(x) - g(x)]\,dx$

Differential Equations

Linear Differential Equations

$\frac{dy}{dx} + p(x)y = q(x) \quad \Rightarrow \quad \mu y = \int \mu q(x)\,dx$, where $\mu = e^{\int p(x)\,dx}$

Constant Coefficient of Homogeneous Equations

Roots of Auxiliary Equation, $r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

General Solutions to the Auxiliary Equation:

2 Real & Unequal Roots ($b^2 - 4ac > 0$)	$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
Repeated Roots ($b^2 - 4ac = 0$)	$y = c_1 e^{rx} + c_2 x e^{rx}$
2 Complex Roots ($b^2 - 4ac < 0$)	$y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

Constant Coefficient of Non-Homogeneous Equations

$y = y_c + y_p$ [y_c: complementary solution, y_p: particular solution]

Vector

Length of Vector

The length of the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Dot Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$

Cross Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

Area for parallelogram *PQRS*

$= \left| \vec{PQ} \times \vec{PR} \right|$

Area for triangle PQR

$= \frac{1}{2} \left| \vec{PQ} \times \vec{PR} \right|$

Equation of Lines

Vector equation: $\mathbf{r} = \mathbf{r}_0 + \mathbf{tv}$

Parametric equations: $x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct$

Symmetric equation: $\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$

Equation of Planes

Vector equation: $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Linear equation: $ax + by + cz + d = 0$

Angle between Two Planes: $\theta = \cos^{-1}\left(\frac{\mathbf{n}_1 \bullet \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|} \right)$